

*Column generation and Benders' Decomposition to maximize the lifetime in connected wireless sensor networks*

Fabian Castaño<sup>1,2</sup>, André Rossi<sup>1</sup>, Marc Sevaux<sup>1</sup>, Nubia Velasco<sup>2</sup>



Université  
de Bretagne-Sud



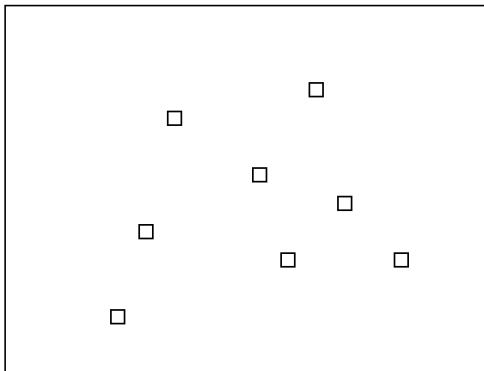
Universidad de  
**los Andes**  
Facultad de Ingeniería

<sup>1</sup>Lab-STICC, Université de Bretagne-Sud, France

<sup>2</sup>PyLO, Universidad de los Andes, Colombia

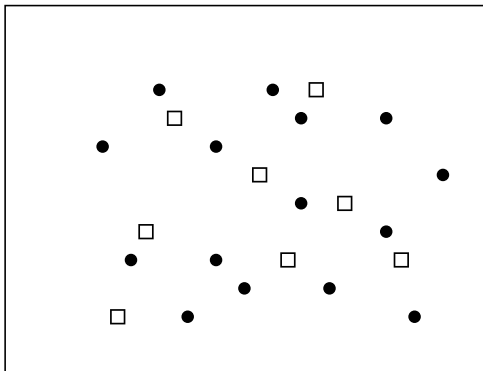
# Maximum network lifetime problem with connectivity and coverage constraints (CMLP)

- ▶ A set of targets  $T$



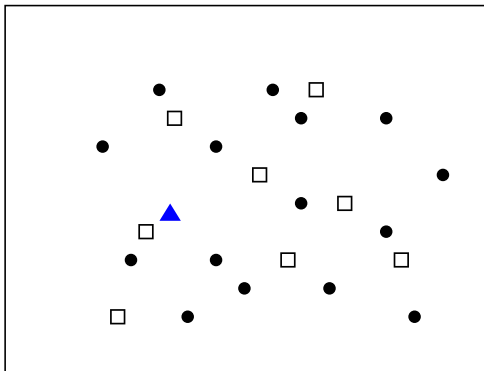
# Maximum network lifetime problem with connectivity and coverage constraints (CMLP)

- ▶ A set of targets  $T$
- ▶ A set of sensors  $S$



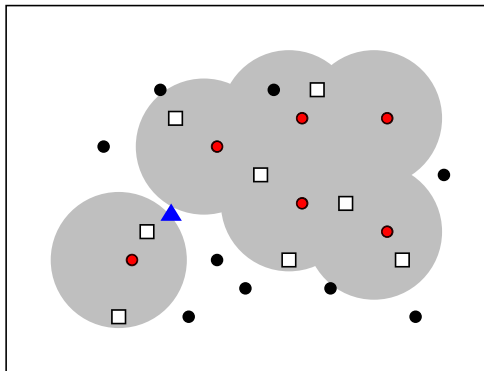
# Maximum network lifetime problem with connectivity and coverage constraints (CMLP)

- ▶ A set of targets  $T$
- ▶ A set of sensors  $S$
- ▶ A base station  $B$



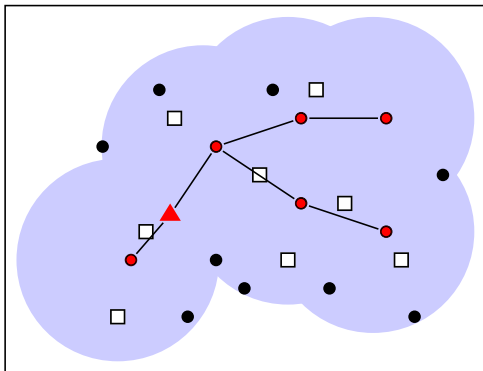
# Maximum network lifetime problem with connectivity and coverage constraints (CMLP)

- ▶ A set of targets  $T$
- ▶ A set of sensors  $S$
- ▶ A base station  $B$
- ▶ Sensing range  $R_s$

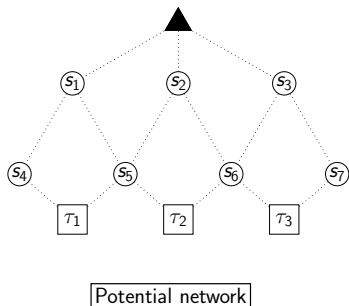


# Maximum network lifetime problem with connectivity and coverage constraints (CMLP)

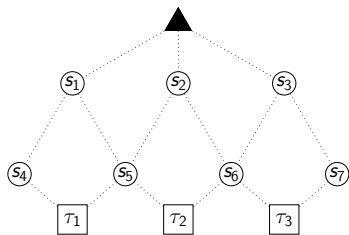
- ▶ A set of targets  $T$
- ▶ A set of sensors  $S$
- ▶ A base station  $B$
- ▶ Sensing range  $R_s$
- ▶ Communication range  $R_c$
- ▶ Cover set
- ▶ Connectivity required



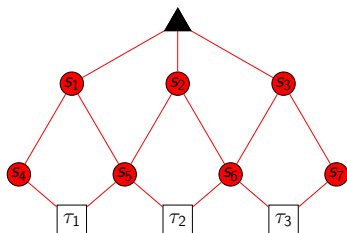
# Strategies to extend network lifetime



# Strategies to extend network lifetime



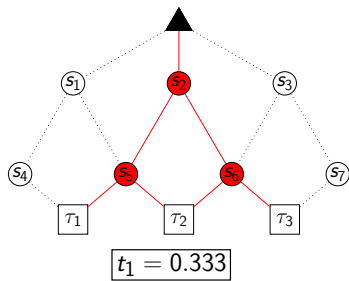
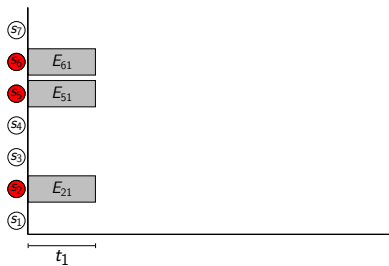
Potential network



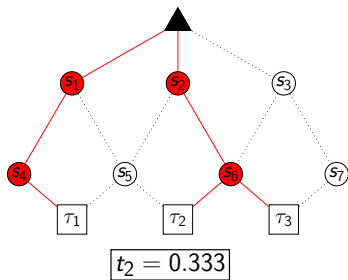
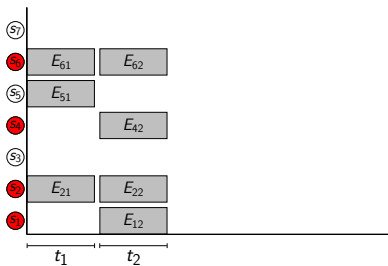
Total Lifetime=1



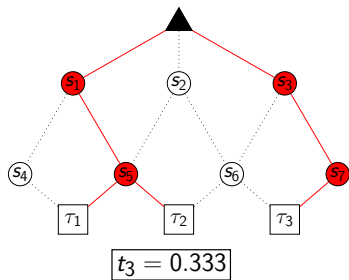
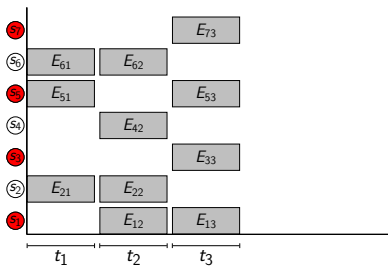
# Strategies to extend network lifetime



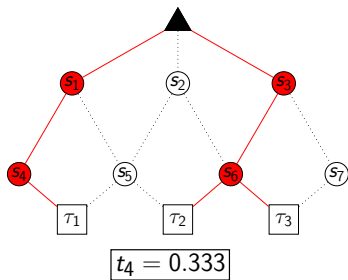
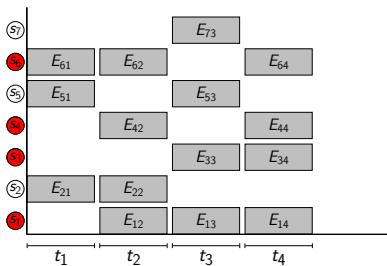
# Strategies to extend network lifetime



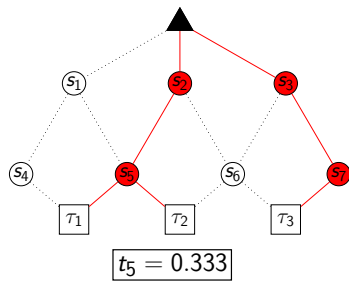
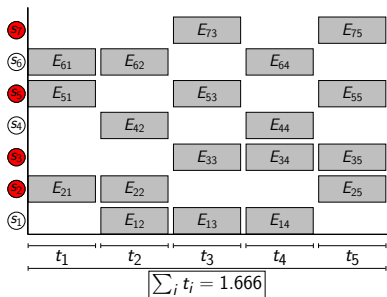
# Strategies to extend network lifetime



# Strategies to extend network lifetime

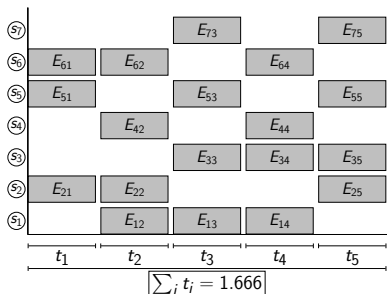


# Strategies to extend network lifetime



# Strategies to extend network lifetime

## Representation



$$\begin{pmatrix} 0 & 0 & E_{73} & 0 & E_{75} \\ E_{61} & E_{62} & 0 & E_{64} & 0 \\ E_{51} & 0 & E_{53} & 0 & E_{55} \\ 0 & E_{42} & 0 & E_{44} & 0 \\ 0 & 0 & E_{33} & E_{34} & E_{35} \\ E_{21} & E_{22} & 0 & 0 & E_{25} \\ 0 & E_{12} & E_{13} & E_{14} & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix}$$

# Lifetime maximization in wireless sensor networks

## Problem representation

## Representation

$$\begin{pmatrix} 0 & 0 & E_{73} & 0 & E_{75} \\ E_{61} & E_{62} & 0 & E_{64} & 0 \\ E_{51} & 0 & E_{53} & 0 & E_{55} \\ 0 & E_{42} & 0 & E_{44} & 0 \\ 0 & 0 & E_{33} & E_{34} & E_{35} \\ E_{21} & E_{22} & 0 & 0 & E_{25} \\ 0 & E_{12} & E_{13} & E_{14} & 0 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix}$$

## Model

Maximize:

$$t_1 + t_2 + t_3 + t_4 + t_5$$

Subject to:

$$E_{73} t_3 + E_{75} t_5 \leq b_7 \quad (1)$$

$$E_{61} t_1 + E_{62} t_2 + E_{64} t_4 \leq b_6 \quad (2)$$

$$E_{51} t_1 + E_{53} t_3 + E_{55} t_5 \leq b_5 \quad (3)$$

$$E_{41} t_2 + E_{44} t_4 \leq b_4 \quad (4)$$

$$E_{33} t_3 + E_{34} t_4 + E_{35} t_5 \leq b_3 \quad (5)$$

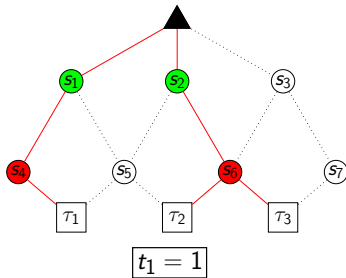
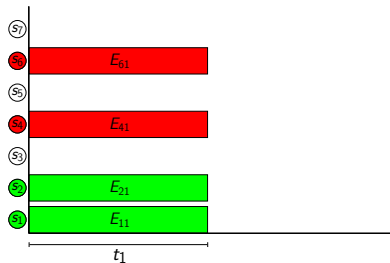
$$E_{21} t_1 + E_{22} t_2 + E_{25} t_5 \leq b_2 \quad (6)$$

$$E_{11} t_2 + E_{13} t_3 + E_{14} t_4 \leq b_1 \quad (7)$$

$$t_1, t_2, t_3, t_4, t_5 \geq 0 \quad (8)$$

# Strategies to extend network lifetime

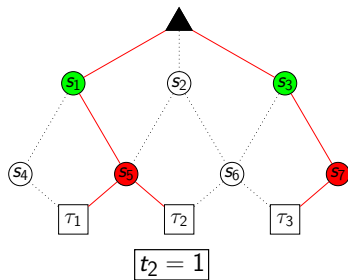
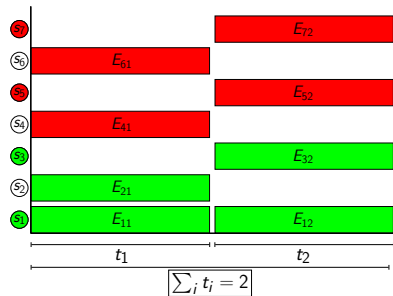
## Multiple role allocation (MR-CMLP)





# Strategies to extend network lifetime

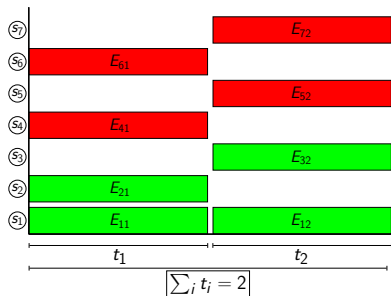
## Multiple role allocation (MR-CMLP)



# Strategies to extend network lifetime

## Multiple role allocation (MR-CMLP)

### Representation



$$\begin{pmatrix} 0 & E_{72} \\ E_{61} & 0 \\ 0 & E_{52} \\ E_{41} & 0 \\ 0 & E_{32} \\ E_{21} & 0 \\ E_{11} & E_{12} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

# Lifetime maximization in wireless sensor networks

## Problem representation

### Representation

$$\begin{pmatrix} 0 & E_{72} \\ E_{61} & 0 \\ 0 & E_{52} \\ E_{41} & 0 \\ 0 & E_{32} \\ E_{21} & 0 \\ E_{11} & E_{12} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

### Model

Maximize:

$$t_1 + t_2$$

Subject to:

$$E_{72} t_2 \leq b$$

$$E_{61} t_1 \leq b$$

$$E_{52} t_2 \leq b$$

$$E_{41} t_1 \leq b$$

$$E_{32} t_2 \leq b$$

$$E_{21} t_1 \leq b$$

$$E_{11} t_1 + E_{12} t_2 \leq b$$

$$t_1, t_2 \geq 0$$

# Lifetime maximization in wireless sensor networks

## General model for MLP

### Representation

$$\begin{pmatrix} E_{51} & E_{52} & E_{53} & E_{54} & E_{55} \\ E_{41} & E_{42} & E_{43} & E_{44} & E_{45} \\ E_{31} & E_{32} & E_{33} & E_{34} & E_{35} \\ E_{21} & E_{22} & E_{23} & E_{24} & E_{25} \\ E_{11} & E_{12} & E_{13} & E_{14} & E_{15} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix}$$

### Model

Maximize:

$$t_1 + t_2 + t_3 + t_4 + t_5$$

Subject to:

$$E_{51} t_1 + E_{52} t_1 + E_{53} t_1 + E_{54} t_1 + E_{55} t_1 \leq b_5$$

$$E_{41} t_1 + E_{42} t_1 + E_{43} t_1 + E_{44} t_1 + E_{45} t_1 \leq b_4$$

$$E_{31} t_1 + E_{32} t_1 + E_{33} t_1 + E_{34} t_1 + E_{35} t_1 \leq b_3$$

$$E_{21} t_1 + E_{22} t_1 + E_{23} t_1 + E_{24} t_1 + E_{25} t_1 \leq b_2$$

$$E_{11} t_1 + E_{12} t_1 + E_{13} t_1 + E_{14} t_1 + E_{15} t_1 \leq b_1$$

$$t_1, t_2, t_3, t_4, t_5 \geq 0$$

# Lifetime maximization in wireless sensor networks

## General model for MLP

### Representation

$$\begin{pmatrix} E_{51} & E_{52} & E_{53} & E_{54} & E_{51} \\ E_{41} & E_{42} & E_{43} & E_{44} & E_{45} \\ E_{31} & E_{32} & E_{33} & E_{34} & E_{35} \\ E_{21} & E_{22} & E_{23} & E_{24} & E_{25} \\ E_{41} & E_{12} & E_{13} & E_{14} & E_{15} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix}$$

### Model

Maximize:

$$\sum_{j|C_j \in \Omega} t_j$$

Subject to:

$$\sum_{j|C_j \in \Omega} E_{s_u j} t_j \leq b_{s_u} \quad \forall s_u \in S$$

$$t_j \geq 0 \quad \forall j|C_j \in \Omega$$

# Maximum network lifetime problem (CMLP)

## Considerations

### Problem decomposition

The problem has been reduced to identify subset of sensors and to create schedules for them.

### Drawback:

- ▶ The number of subsets is too large



# Maximum network lifetime problem (CMLP)

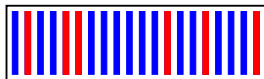
## Considerations

### Problem decomposition

The problem has been reduced to identify subset of sensors and to create schedules for them.

### Drawback:

- ▶ The number of subsets is too large
- ▶ Only part of them will be in the optimal solution



# Maximum network lifetime problem (CMLP)

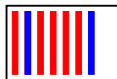
## Considerations

### Problem decomposition

The problem has been reduced to identify subset of sensors and to create schedules for them.

### Drawback:

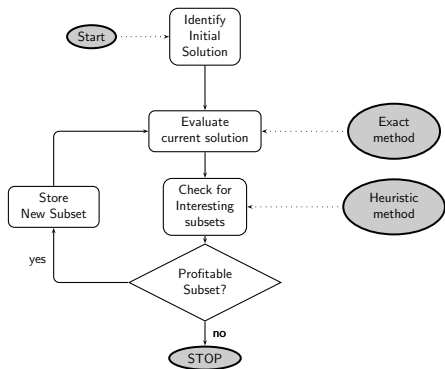
- ▶ The number of subsets is too large
- ▶ Only part of them will be in the optimal solution
- ▶ Gradually generate them is a more efficient strategy





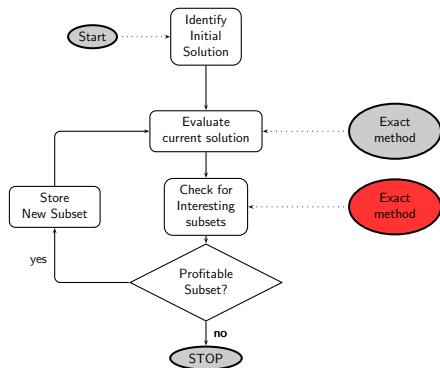
# Iterative improvement of lifetime

## Column generation algorithm



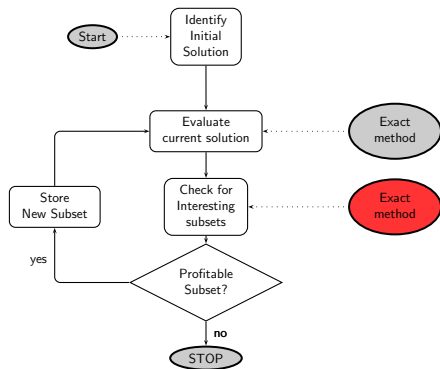
# Iterative improvement of lifetime

## Column generation algorithm



# Iterative improvement of lifetime

## Column generation algorithm

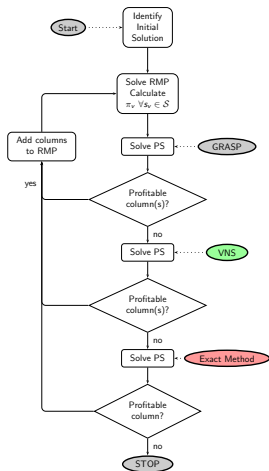


### Check for subsets?

- ▶ Set covering
- ▶ Facility location
- ▶ Minimum Steiner tree
- ▶ Minimum power multicast tree
- ▶ Node weighted Steiner tree

# Efficient implementations?

## Hybrid approaches

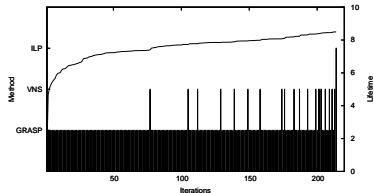
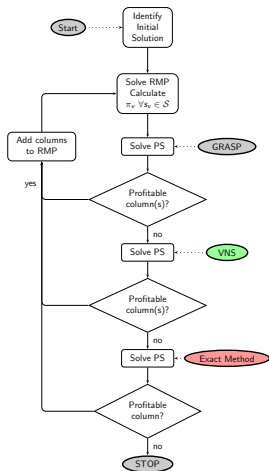


## Ideas (Castaño *et al.*, 2013)

- Use fast heuristic approaches to solve pricing subproblem as much as possible
- Improve the performance of the exact approaches by using advanced techniques

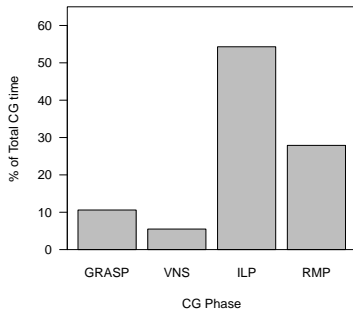
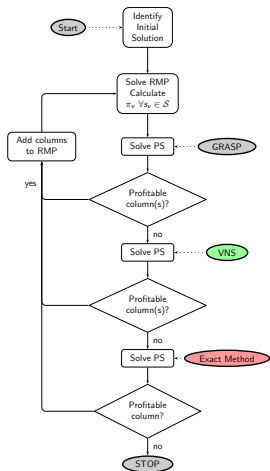
# Efficient implementations?

## Hybrid approaches



# Efficient implementations?

## Hybrid approaches



# Decomposition based approach

## General approach

---

Restricted Master Problem (Scheduling decisions)

---

$$\text{Maximize: } \sum_{j|C_j \in \Omega} t_j$$

$$\sum_{j|C_j \in \Omega} E_{s_u} t_j \leq b_{s_u} \quad \forall s_u \in S$$

$$t_j \geq 0 \quad \forall j|C_j \in \Omega$$

---

---

# Decomposition based approach

## General approach

---

### Restricted Master Problem (Scheduling decisions)

---

$$\begin{aligned}
 & \text{Maximize: } \sum_{j|C_j \in \Omega} t_j \\
 & \sum_{j|C_j \in \Omega} E_{s_u} t_j \leq b_{s_u} \quad \forall s_u \in S \\
 & t_j \geq 0 \quad \forall j|C_j \in \Omega
 \end{aligned}$$


---

## Considerations

- ▶  $\Omega$ : set of feasible covers
- ▶  $|\Omega|$ : Exponential (number of variables)
- ▶ Use a reduced set  $\Omega' \subseteq \Omega$



# Decomposition based approach

## General approach

---

### Restricted Master Problem (Scheduling decisions)

---

$$\begin{aligned} \text{Maximize: } & \sum_{j|C_j \in \Omega} t_j \\ \sum_{j|C_j \in \Omega} E_{s_{ij}} t_j & \leq b_{s_u} \quad \forall s_u \in S \\ t_j & \geq 0 \quad \forall j|C_j \in \Omega \end{aligned}$$


---

---

### Pricing subproblem (MR-CMLP)

---

$$\text{Maximize : } 1 - \sum_{v \in S} \sum_{l \in \mathcal{E}} l y_{vlj} \pi_v$$

Subject to:

$$\begin{aligned} \sum_{u \in \mathcal{S}|\exists a(v,u)} x_{vu} - \sum_{u \in \mathcal{S}|\exists a(u,v)} x_{uv} &= 1 \quad \forall v \in \mathcal{T} \\ \sum_{u \in \mathcal{N}|\exists a(v,u)} x_{vu} - \sum_{u \in \mathcal{N}|\exists a(u,v)} x_{uv} &= 0 \quad \forall v \in \mathcal{S} \\ \sum_{u \in \mathcal{N}|\exists a(S_0,u)} x_{S_0u} - \sum_{u \in \mathcal{N}|\exists a(u,S_0)} x_{uS_0} &= -|\mathcal{T}| \\ x_{uv} &\leq |\mathcal{T}|(y_{vE_{ij}} + y_{vE_{ji}}) \quad \forall u, v \in \mathcal{S}|\exists a(u, v) \\ x_{vu} &\leq |\mathcal{T}|(y_{vE_{ij}} + y_{vE_{ji}}) \quad \forall v, u \in \mathcal{S}|\exists a(v, u) \\ x_{uv} &\leq y_{vE_{ij}} \quad \forall u \in \mathcal{T}, \forall v \in \mathcal{S}|\exists a(u, v) \\ \sum_{l \in \mathcal{E}} y_{vlj} &= 1 \quad \forall v \in \mathcal{S} \\ x_{uv} &\in \mathbb{Z}^+ \cup \{0\} \quad \forall u, v \in \mathcal{N} \\ y_{vlj} &\in \{0, 1\} \quad \forall v \in \mathcal{N}, l \in \mathcal{E} \end{aligned}$$

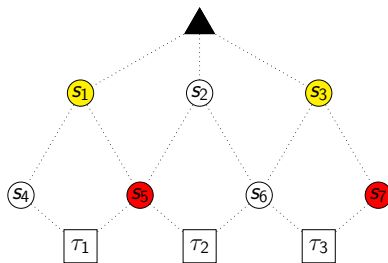

---

# Column generation approach

## Strategies to solve PS

### Facts

- ▶ If sensor role were fixed, resulting PS is a network flow problem

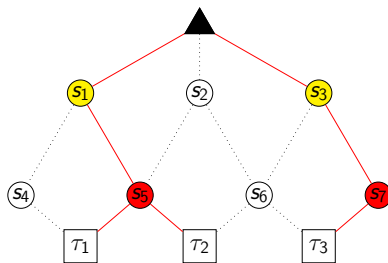


# Column generation approach

## Strategies to solve PS

### Facts

- ▶ If sensor role were fixed, resulting PS is a network flow problem
- ▶ Feasibility is easy to check

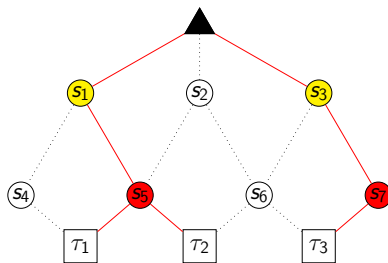


# Column generation approach

## Strategies to solve PS

### Facts

- ▶ If sensor role were fixed, resulting PS is a network flow problem
- ▶ Feasibility is easy to check
- ▶ Connectivity and coverage can be solved at different stages



# Column generation approach

## Bender's reformulation

Consider the problem ...

$$z = \min cx + hy$$

$$Ax + Gy \leq b$$

$$x \in \mathbb{R}_+^n, y \in Y$$

Remarks:

- ▶  $(y \in Y)$  are complicating variables

# Column generation approach

## Bender's reformulation

### Linear version

$$z_{LP} = \min cx$$

$$Ax \leq b - Gy$$

$$x \in \mathbb{R}_+^n$$

### Remarks:

- ▶  $(y \in Y)$  are complicating variables
- ▶ When variables  $(y \in Y)$  are fixed

# Column generation approach

## Bender's reformulation

### Associated dual

$$\max u(b - Gy)$$

$$uA \geq c$$

$$u \in \mathbb{R}_+^n$$

### Remarks:

- ▶  $(y \in Y)$  are complicating variables
- ▶ When variables  $(y \in Y)$  are fixed
- ▶ Extreme points:  
 $\{u^k \in \mathbb{R}_+^m : k \in K\}$
- ▶ Extreme rays:  
 $\{v^d \in \mathbb{R}_+^m : d \in D\}$

# Column generation approach

## Bender's reformulation

### New problem reformulation

$$z = \min(hy + \max_{k \in K} u^k(b - Gy))$$

$$v^d(b - Gy) \geq 0 \quad \forall d \in D$$

$$y \in Y \subseteq \mathbb{Z}_+^n$$

- ▶ If cost associated to “easy variables” is zero

$$z = \min(hy)$$

$$v^d(b - Gy) \geq 0 \quad \forall d \in D$$

$$y \in Y \subseteq \mathbb{Z}_+^n$$



# Column generation approach

Solution approach for Bender's reformulation

## Remarks

- ▶  $\mathcal{D}$  is a set of constraints (exponential size)
- ▶ Only a subset of the cut constraints will be active in the optimal solution
- ▶ A cut generation algorithm can be adapted to identify interesting cuts

# Column generation approach

Solution approach for Bender's reformulation

## Remarks

- ▶  $\mathcal{D}$  is a set of constraints (exponential size)
- ▶ Only a subset of the cut constraints will be active in the optimal solution
- ▶ A cut generation algorithm can be adapted to identify interesting cuts

## Cut generation algorithm

1.  $y \leftarrow \bar{y}$ ,  $Upper\_Bound \leftarrow f(\bar{y})$
2. Solve Bender's Master  $Z_M$ ,  
 $Lower\_Bound \leftarrow Z_M^*$
3. Solve  $Z_{LP}$  with fixed  $\bar{y}$
- 4a. If  $Z_{LP}$  is feasible:
  - ▶  $Upper\_Bound \leftarrow Z_{LP}^*$ , Add Bender's cut
- 4b. Else if  $Z_{LP}$  is Infeasible:
  - ▶ Obtain extreme ray (by retrieving Farkas duals) and add feasibility cut
5. If  $Upper\_Bound = Lower\_Bound$ , stop. Otherwise, go to step 2

# Column generation approach

## Bender's reformulation of PS

---

### Pricing subproblem (MR-CMLP)

---

$$\text{Maximize : } 1 - \sum_{v \in \mathcal{S}} \sum_{l \in \mathcal{E}} l y_{vlj} \pi_v$$

Subject to:

$$\sum_{u \in \mathcal{S} | \exists a(v,u)} x_{vu} - \sum_{u \in \mathcal{S} | \exists a(u,v)} x_{uv} = 1 \quad \forall v \in \mathcal{T}$$

$$\sum_{u \in \mathcal{N} | \exists a(v,u)} x_{vu} - \sum_{u \in \mathcal{N} | \exists a(u,v)} x_{uv} = 0 \quad \forall v \in \mathcal{S}$$

$$\sum_{u \in \mathcal{N} | \exists a(s_0,u)} x_{s_0 u} - \sum_{u \in \mathcal{N} | \exists a(u,s_0)} x_{us_0} = -|\mathcal{T}|$$

$$x_{uv} \leq |\mathcal{T}| (y_{vE,j} + y_{vE,i}) \quad \forall u, v \in \mathcal{S} | \exists a(u, v)$$

$$x_{vu} \leq |\mathcal{T}| (y_{vE,j} + y_{vE,i}) \quad \forall v, u \in \mathcal{S} | \exists a(v, u)$$

$$x_{uv} \leq y_{vE,j} \quad \forall u \in \mathcal{T}, \forall v \in \mathcal{S} | \exists a(u, v)$$

$$\sum_{l \in \mathcal{E}} y_{vlj} = 1 \quad \forall v \in \mathcal{S}$$

$$x_{uv} \in \mathbb{Z}^+ \cup \{0\} \quad \forall u, v \in \mathcal{N}$$

$$y_{vlj} \in \{0, 1\} \quad \forall v \in \mathcal{N}, l \in \mathcal{E}$$


---

---

### Benders Reformulation (MR-CMLP)

---

$$\min Z_{BMP} = \sum_{v \in \mathcal{S}} \sum_{l \in \mathcal{E}} y_{vlj} l \pi_v$$

Subject to:

$$\sum_{v \in \mathcal{S}} \theta_v^d - |\mathcal{T}| \theta_B^d + \sum_{u \in \mathcal{N}, v \in \mathcal{N}} (y_{vE,j} + y_{vE,i}) \beta_{uv}^d |\mathcal{T}| +$$

$$\sum_{v \in \mathcal{N}, u \in \mathcal{N}} (y_{vE,j} + y_{vE,i}) \beta_{vu}^d |\mathcal{T}| + \sum_{u \in \mathcal{N}, v \in \mathcal{N}} (y_{vE,j}) \eta_{uv}^d \geq 0 \quad \forall d \in \mathbb{D}$$

$$\sum_{l \in \mathcal{E}} y_{vlj} = 1 \quad \forall v \in \mathcal{S}$$

$$y_{vlj} \in \{0, 1\} \quad \forall v \in \mathcal{N}, \forall l \in \mathcal{E}$$

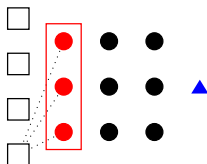

---

# Column generation approach

Valid inequalities to improve convergence

## Coverage

$$\sum_{v \in \mathcal{S} | \exists a(u,v)} y_{vE_{sj}} \geq 1 \quad \forall u \in \mathcal{T}$$

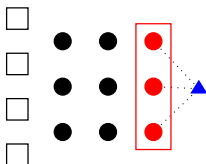


# Column generation approach

Valid inequalities to improve convergence

## Base Station connectivity

$$\sum_{v \in \mathcal{S} | \exists a(v, S_0)} (y_{vE_rj} + y_{vE_{Sj}}) \geq 1$$

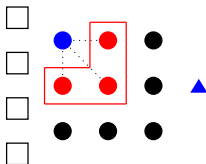


# Column generation approach

Valid inequalities to improve convergence

No isolation

$$\sum_{v|\exists a(u,v)} (y_{vE_rj} + y_{vE_{Sj}}) \geq (y_{uE_rj} + y_{uE_{Sj}}) \quad \forall u \in \mathcal{S} \setminus \{a(u, S_0)\}$$

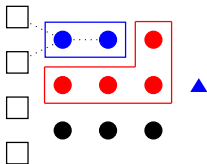


# Column generation approach

Valid inequalities to improve convergence

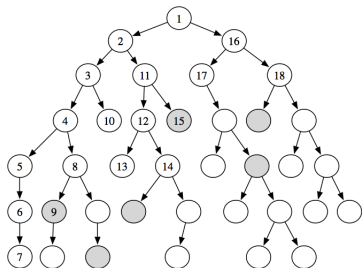
## Single connected component

$$\sum_{s_u \in CC_i} (y_{uE_rj} + y_{uE_{Sj}}) \leq |CC_i| \sum_{s_u \in CC_i, s_v \in \mathcal{S} \setminus CC_i | \exists a(u,v)} (y_{vE_rj} + y_{vE_{Sj}}) \quad \forall CC_i \in \mathcal{CC}$$



# Column generation approach

## Further improvements



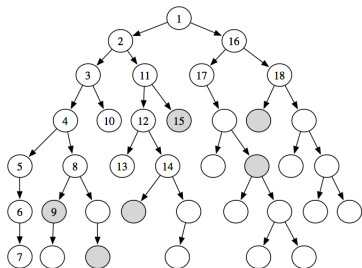
## Improvements

- Embed cuts within a Branch & Bound



# Column generation approach

## Further improvements



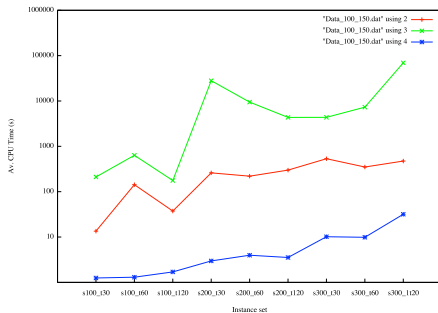
## Improvements

- ▶ Embed cuts within a Branch & Bound
- ▶ Complementary connectivity constraints

# Computational results

A comparison of Bender's decomposition and ILP to solve PS

Up to 1000 less time is consumed to solve PS.



# Computational results

Comparison of strategies used to solve efficiently the CG approach

Instance	S	K	R <sub>0</sub>	ILP Solver				BBC				
				Opt	Sol	Time	n iters	GAP	Sol	Time	n iters	GAP
CMLP_MR_001	100	15	100	5,40	1,00	3600,0	2	81%	5,40	600,6	76	0%
CMLP_MR_002	100	15	100	4,00	4,00	119,2	43	0%	4,00	21,7	32	0%
CMLP_MR_003	100	15	100	3,75	3,50	63,3	24	7%	3,75	485,3	127	0%
CMLP_MR_004	100	15	100	4,00	4,00	214,1	31	0%	4,00	26,8	35	0%
CMLP_MR_005	100	30	100	5,00	4,30	3600,0	40	14%	5,00	3,9	108	0%
CMLP_MR_006	100	30	100	4,00	4,00	99,2	42	0%	4,00	2,9	71	0%
CMLP_MR_007	100	30	100	3,00	3,00	66,2	24	0%	3,00	2,6	48	0%
CMLP_MR_008	100	30	100	4,00	4,00	73,6	32	0%	4,00	3,0	70	0%
CMLP_MR_009	200	15	100	8,00	1,00	3600,0	2	88%	8,00	29,8	88	0%
CMLP_MR_010	200	15	100	12,00	5,23	3600,0	22	56%	12,00	78,2	105	0%
CMLP_MR_011	200	15	100	9,00	1,00	3600,0	2	89%	9,00	35,4	137	0%
CMLP_MR_012	200	15	100	14,60	2,00	3600,0	5	86%	10,86	3600,0	137	26%
CMLP_MR_013	200	30	100	8,00	8,00	485,6	63	0%	8,00	23,5	159	0%
CMLP_MR_014	200	30	100	7,00	7,00	225,4	46	0%	7,00	16,3	133	0%
CMLP_MR_015	200	30	100	9,00	1,00	3600,0	1	89%	9,00	44,8	157	0%
CMLP_MR_016	200	30	100	11,40	10,81	3600,0	173	5%	10,07	3600,0	347	12%
CMLP_MR_017	300	15	100	15,00	2,00	3600,0	6	87%	15,00	76,9	183	0%
CMLP_MR_018	300	15	100	21,25	1,00	3600,0	1	95%	18,16	3600,0	1342	15%
CMLP_MR_019	300	15	100	12,00	1,00	3600,0	1	92%	12,00	114,8	516	0%
CMLP_MR_020	300	15	100	16,00	4,00	3600,0	13	75%	16,00	153,5	144	0%
CMLP_MR_021	300	30	100	15,00	1,00	3600,0	1	93%	15,00	153,7	368	0%
CMLP_MR_022	300	30	100	13,20	13,00	2141,6	109	2%	13,20	223,7	866	0%
CMLP_MR_023	300	30	100	12,00	12,00	3600,0	92	0%	12,00	63,5	163	0%
CMLP_MR_024	300	30	100	13,00	1,00	3600,0	1	92%	13,00	120,2	413	0%
CMLP_MR_025	400	15	100	18,60	1,00	3600,0	2	95%	18,60	2619,1	4104	0%
CMLP_MR_026	400	15	100	18,00	1,00	3600,0	3	94%	18,00	168,5	792	0%
CMLP_MR_027	400	15	100	22,00	12,36	3600,0	57	44%	22,00	64,0	280	0%
CMLP_MR_028	400	15	100	17,00	1,00	3600,0	1	94%	17,00	202,0	898	0%
CMLP_MR_029	400	30	100	17,60	1,00	3600,0	1	94%	17,60	1600,4	1916	0%
CMLP_MR_030	400	30	100	18,00	1,00	3600,0	1	94%	18,00	205,1	789	0%
CMLP_MR_031	400	30	100	21,00	1,00	3600,0	1	95%	21,00	166,6	575	0%
CMLP_MR_032	400	30	100	17,00	1,00	3600,0	1	94%	17,00	190,3	725	0%
Number of optimals				8				29				

## Conclusions & future work

- ▶ A column generation strategy combined with an efficient branch and cut based on benders decomposition
- ▶ The experiments indicate that the exact approach improves previous results obtained with hybrid approaches
- ▶ The method can be “easily” adapted to cover a wide range of WSN problems.
- ▶ Enhancements with (meta)heuristic approaches are expected to be successful in more sophisticated WSN models

Visit us ...



OR-Group@Lab-STICC

<http://or-labsticc.univ-ubs.fr>

